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Nonperturbative Condensates in the Electroweak Phase-Transition [1]

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Abstract

We discuss the electroweak phase-transition in the early universe, using non-perturbative flow equations for a computation of the free energy. For a scalar mass above ~ 70 GeV, high-temperature perturbation theory cannot describe this transition reliably. This is due to the dominance of three-dimensional physics at high temperatures which implies that the effective gauge coupling grows strong in the symmetric phase. We give an order of magnitude-estimate of non-perturbative effects in reasonable agreement with recent results from electroweak lattice simulations.

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1. High Temperature Phase-Transitions

There has been a lively interest in phase-transitions in gauge-theories over the last decades since the original work by Kirzhnits and Linde [1], indicating that spontaneously broken symmetries are restored at high temperatures. The most prominent examples are the electroweak phase-transition, i.e. the restauration of the $SU(2)_L \times U(1)_Y$-symmetry of the standard-model, and the transition in QCD where (approximate) chiral symmetry is restored. These transitions are of interest for different reasons. In the case of the electroweak phase-transition the most prominent question is whether the observed baryon-asymmetry could have been produced during such a transition in the early universe [2]. For this scenario to work, one needs besides CP-violation also a sufficiently large deviation of baryon-number-violating processes from thermal equilibrium [3]. This translates into the requirement that the transition be strong enough first order. On the other hand, if in the standard model the transition is a second or only weakly first order one, or even an analytical crossover instead of a true phase transition, an extension of the standard model is needed. Then either a $B - L$ asymmetry could be generated in the early universe or the electroweak phase transition could become sufficiently strongly first order [2]. An answer to the question if the observed baryon-asymmetry originated in the electroweak phase-transition requires a detailed understanding of the dynamics of the transition.

The interest in the chiral transition in QCD is more directly related to speculations about the possibility of experimental access to the high-temperature state of QCD at heavy-ion colliders. At present, there is no convincing evidence that such a state has already been detected. A better understanding of the high temperature phase would certainly be helpful in answering the question of clean experimental signatures [4].

For QCD there was never any question about the need for non-perturbative methods in examining the details of the transition. Correspondingly there are a host of studies with the help of lattice-simulations or effective models. In the case of the electroweak transition the hope that a description by means of high-temperature perturbation theory might be sufficient prevailed for some time. For the most prominent qualitative features this view seems justified for a small mass of the Higgs scalar. For a realistic scalar mass above the experimental bound it has been argued, however, that a quantitative description of the high-temperature behavior and the phase-transition is only possible by non-perturbative methods since strong effective couplings are involved [5-8]. This holds despite the fact that the zero temperature electroweak interactions are weak.

The deeper reason for the breakdown of perturbation theory lies in the effective three-dimensional character of the high-temperature field theory [9]. Field theory at nonvanishing temperature T can be formulated in terms of an Euclidean functional integral where the "time dimension" is compactified on a torus with radius T^{-1}. For phenomena at distances larger than T^{-1} the Euclidean time dimension

cannot be resolved. Integrating over modes with momenta $p^2 > (2\pi T)^2$ or, alternatively, over the higher Fourier modes on the torus (the $n \neq 0$ Matsubara frequencies) leads to "dimensional reduction" to an effective three-dimensional theory. This is very similar to dimensional reduction in Kaluza-Klein theories [10] for gravity. The change of the effective dimensionality for distances larger than T^{-1} is manifest in the renormalization group approach [5] for a computation of the temperature-dependent effective potential or free energy. Here one integrates over all fluctuations with momenta $p^2 > k^2$ and follows the dependence of the effective potential on the infrared scale k, finally letting $k \to 0$. The scale dependence of the effective renormalized couplings is governed by the usual perturbative β-functions only for $k^2 > (2\pi T)^2$. In contrast, for $k^2 < (2\pi T)^2$ the running of the couplings was found to be determined by three-dimensional β-functions instead of the perturbative four-dimensional ones[1]. As an alternative to integrating out all modes with $p^2 > (2\pi T)^2$ an effective three-dimensional theory for the long distance electroweak physics is also obtained [12, 13] by integrating out the higher Matsubara frequencies[2].

If the three-dimensional running of the couplings becomes important, the physics of the phase-transition is dominated by classical statistics even in case of a quantum field theory. A second order phase-transition is characterized by an infinite correlation length. The critical exponents which describe the behavior near the critical temperature are always those of the corresponding classical statistical system. Since the fixpoints of the three-dimensional β-functions are very different from the four-dimensional (perturbative) fixpoints, we conclude that high-temperature perturbation theory is completely misleading in the vicinity of a second order phase-transition. This argument extends to sufficiently weak first order transitions. A second related example for the breakdown of perturbation theory is the symmetric phase of the electroweak gauge theory. The gauge bosons are massless in perturbation theory and the three-dimensional running always dominates at large distances [6].

In order to understand the high-temperature behavior of a theory we should understand the qualitative features of the β-functions in three dimensions. These β-functions have nothing to do with the ultraviolet regularization of the field theory - in this respect there is no difference between vanishing and nonvanishing temperature. They are rather related to the infrared behavior of the theory or the dependence of Green functions on some sort of infrared cutoff. According to Wilson's concept of the renormalization group these β-functions describe the scale dependence of the couplings if one looks at the system on larger and larger distances. For an understanding of systems with approximate scaling in a certain range it is useful to define dimensionless couplings. One divides out an appropriate power of the infrared cutoff k which plays the role of the renormalization scale. For example, the gauge coupling g in the effective three-dimensional theory is related to the four-dimensional coupling

[1] Related arguments in a different context can be found in [11].

[2] For an earlier treatment of dimensional reduction in high-temperature QCD see [14]

g_4 and the temperature by

$$g^2 = \frac{\bar{g}_3^2}{k} = g_4^2 \frac{T}{k} \tag{1}$$

For the $SU(2)$-Higgs model in three dimensions relevant for the electroweak phase transition, the dependence of g^2 on the scale k is given[3] by $\beta_{\bar{g}_3^2} = \frac{\partial \bar{g}_3^2}{\partial t} = -\frac{23}{24\pi k}\bar{g}_3^4 - \ldots$ with $t = \ln k$ [6]. One concludes that a non-abelian gauge theory like the electroweak theory is confining also in three dimensions. We have depicted the running of the

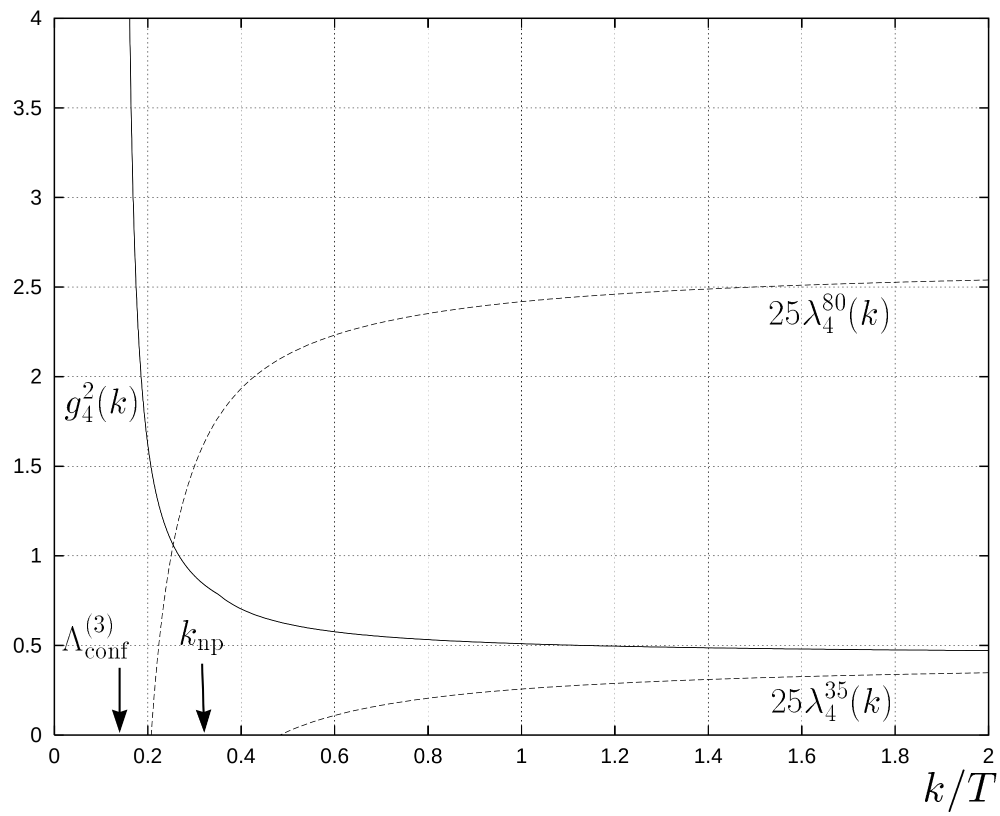

Figure 1: The running of the four-dimensional couplings g_4^2 (solid line) and λ_4 (dashed lines). The initial values of λ_4 correspond to scalar masses of 35 and 80 GeV respectively.

four-dimensional gauge coupling g_4^2 and the four-dimensional quartic scalar coupling λ_4 in fig. 1. The deviation of $g_4^2(k)$ from the zero temperature value $g_4^2 = 4/9$ can be interpreted as a measure for the validity of the one-loop approximation. For sufficiently small initial λ_4 (small physical Higgs boson mass) $\lambda_4(k)$ reaches zero for

[3] The coefficients depend on the precise choice of the infrared cutoff.

k much larger than the three-dimensional confinement scale. One then expects a first-order transition which is analogous to the four-dimensional Coleman-Weinberg scenario [15]. Typical mass scales are of the order k_{cw} where $\lambda(k_{cw}) = 0$. In this case it is expected that high-temperature perturbation theory gives reliable results. On the other hand, if the three-dimensional confinement scale $\Lambda^{(3)}_{\rm conf}$ (the value of k for which the gauge coupling diverges or becomes very large) is reached with $\lambda(\Lambda^{(3)}_{\rm conf}) > 0$ the behavior near the phase-transition is described by a strongly interacting electroweak theory. Then strong effective coupling constants appear not only in the symmetric phase, but also in the phase with spontaneous symmetry breaking.

In any case, for a calculation of interesting quantities of a first order transition such as the critical temperature, the nucleation rate of critical bubbles, the surface tension, the latent heat etc. information about the strongly interacting symmetric phase is needed. In view of this fact, there have been different approaches to the electroweak phase-transition by means of lattice simulations [16, 17], gap-equations [18], effective models and the application of renormalization group concepts [6, 8]. In the following we want to review this last approach and present some comparison with results from 3-dimensional lattice simulations.

2. The Average Action

A useful tool for describing the running of couplings in arbitrary dimension is the average action [19]. Consider a simple model with a real scalar field χ. The average scalar field is easily defined by

$$\phi_k(x) = \int d^d y f_k(x-y)\chi(y) \tag{2}$$

with f_k decreasing rapidly for $(x-y)^2 > k^{-2}$ and properly normalized. The average is taken over a volume of size $\sim k^{-d}$. The average action $\Gamma_k[\varphi]$ obtains then by functional integration of the "microscopic variables" χ with a constraint forcing $\phi_k(x)$ to equal the "macroscopic field" $\varphi(x)$ up to small fluctuations. It is the effective action for averages of fields and therefore the analogue in continuous space of the block spin action [20] on the lattice. All modes with momenta $q^2 > k^2$ are effectively integrated out. Lowering k permits to explore the theory at longer and longer distances. The average action has the same symmetries as the original action. As usual it may be expanded in derivatives, with average potential $U_k(\rho), \rho = \frac{1}{2}\varphi^2$, kinetic term, etc.

$$\Gamma_k = \int d^d x \left\{ U_k(\rho) + \frac{1}{2} Z_k(\rho)\partial_\mu\varphi\partial^\mu\varphi + ... \right\} . \tag{3}$$

In a suitable formulation [21] the effective average action corresponds to the generating functional for 1PI Green functions with an infrared cutoff set by the scale k. It interpolates between the classical action for $k \to \infty$ and the effective action for

$k \to 0$. In this version an exact nonperturbative evolution equation [21] describes the dependence of Γ_k on the infrared cutoff k ($t = \ln k$)

$$\frac{\partial}{\partial t}\Gamma_k = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\left(\Gamma_k^{(2)} + R_k\right)^{-1}\frac{\partial}{\partial t}R_k \ . \tag{4}$$

Here $R_k(q)$ is a suitable infrared cutoff which may depend on q^2, as, for example, $R_k = q^2 \exp\left(-q^2/k^2\right)\left(1 - \exp\left(-q^2/k^2\right)\right)^{-1}$ or $R_k = k^2$. The two-point function $\Gamma_k^{(2)}$ obtains by second functional variation of Γ_k

$$\Gamma_k^{(2)}(q', q) = \frac{\delta^2 \Gamma_k}{\delta\varphi(-q')\delta\varphi(q)} \ . \tag{5}$$

Therefore $(\Gamma_k^{(2)} + R_k)^{-1}$ is the exact propagator in presence of the infrared cutoff R_k and the flow equation (4) takes the form of the scale variation of a renormalization group-improved one-loop expression [22]. We emphasize that the evolution equation is fully nonperturbative and no approximations are made. A simple proof can be found in [21]. The exact flow equation (4) can be shown to be equivalent with earlier versions of "exact renormalization group equations" [23] and it encodes the same information as the Schwinger-Dyson equations [24].

An exact nonperturbative evolution equation is not yet sufficient for an investigation of nonperturbative problems like high-temperature field theories. It is far too complicated to be solved exactly. For practical use it is crucial to have a formulation that allows to find reliable nonperturbative approximative solutions. Otherwise speaking, one needs a description of Γ_k in terms of only a few k-dependent couplings. The flow equations for these couplings can then be solved numerically or by analytical techniques. It is on the level of such truncations of the effective average action that suitable approximations have to be found. In this respect the formulation of the effective average action offers important advantages: The average action has a simple physical interpretation and eq. (4) is close to perturbation theory if the couplings are small. The formulation is in continuous space and all symmetries - including chiral symmetries or gauge symmetries [6] - can be respected. Since Γ_k has a representation as a functional integral, alternative methods (different from solutions of the flow equations) can be used for an estimate of its form. Furthermore, the flow equation (4) is directly sensitive to the relevant infrared physics since the contribution of particles with mass larger than k is suppressed by the propagator on the r.h.s. of eq. (4). The closed form of this equation does not restrict one a priori to given expansions like in 1PI n-point functions. In addition the momentum integrals in eq. (4) are both infrared and ultraviolet convergent if a suitable cutoff R_k is chosen. Only modes in the vicinity of $q^2 = k^2$ contribute substantially. This feature is crucial for gauge theories where the formulation of a gauge-invariant ultraviolet cutoff is difficult without dimensional regularization.

3. The Running Gauge Coupling

We are now ready to discuss the running of the three-dimensional gauge coupling. We start from the effective average action for a pure $SU(N_c)$ Yang-Mills theory. It is a gauge-invariant functional of the gauge field A and obeys the exact evolution equation [6] (with Tr including a momentum integration)

$$\frac{\partial}{\partial t}\Gamma_k[A] = \frac{1}{2}\mathrm{Tr}\left\{\frac{\partial R_k[A]}{\partial t}\left(\Gamma_k^{(2)}[A] + \Gamma_k^{\mathrm{gauge}(2)}[A] + R_k[A]\right)^{-1}\right\} - \epsilon_k[A] \ . \tag{6}$$

Here $\Gamma_k^{\mathrm{gauge}(2)}[A]$ is the contribution from a generalized gauge-fixing term in a covariant background gauge and $\epsilon_k[A]$ is the ghost contribution [6]. The infrared cutoff R_k is in general formulated in terms of covariant derivatives. We make the simple truncation

$$\begin{aligned}\Gamma_k[A] &= \frac{1}{4}\int d^dx Z_{F,k} F_{\mu\nu}F^{\mu\nu} \\ \Gamma_k^{\mathrm{gauge}}[A,\bar{A}] &= \frac{1}{2\alpha}\int d^dx Z_{F,k}(D_\mu[\bar{A}](A^\mu - \bar{A}^\mu))^2\end{aligned} \tag{7}$$

with background field $\bar{A}$ and $\Gamma_k^{\mathrm{gauge}(2)}[A] = \Gamma_k^{\mathrm{gauge}(2)}[A, \bar{A} = A]$. In d dimensions the gauge coupling $\hat{g}$ appearing in $F_{\mu\nu}$ and D_μ is a constant independent of k. The effective k-dependent coupling can be associated with the dimensionless renormalized gauge coupling

$$g^2(k) = k^{d-4}\bar{g}_d^2(k) = k^{d-4}Z_{F,k}^{-1}\hat{g}^2 \ . \tag{8}$$

The running of g^2 is related to the anomalous dimension η_F

$$\begin{aligned}\eta_F &= -\frac{\partial}{\partial t}\ln Z_{F,k} \\ \frac{\partial g^2}{\partial t} &= \beta_{g^2} = (d-4)g^2 + \eta_F g^2 \ .\end{aligned} \tag{9}$$

Evaluating (6) for configurations with constant magnetic field and $\alpha = 1$ it was found [6] to obey approximately[4]

$$\frac{\partial g^2}{\partial t} = (d-4)g^2 - \frac{\frac{44}{3}N_c v_d a_d g^4}{1 - \frac{20}{3}N_c v_d b_d g^2} \tag{10}$$

with

$$\begin{aligned}v_d^{-1} &= 2^{d+1}\pi^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right) \\ a_d &= \frac{(26-d)(d-2)}{44}n_1^{d-4} \\ b_d &= \frac{(24-d)(d-2)}{40}l_1^{d-2} \ .\end{aligned} \tag{11}$$

[4] In lowest order in the ϵ-expansion [25] the denominator in the last term is absent and $v_3 a_3$ is replaced by $v_4 a_4$.

Only the momentum integrals ($x \equiv q^2$)

$$
\begin{aligned}
n_1^d &= -\frac{1}{2} k^{-d} \int_0^\infty dx\; x^{\frac{d}{2}} \frac{\partial}{\partial t} \left(\frac{\partial P}{\partial x} P^{-1} \right) \\
l_1^d &= -\frac{1}{2} k^{2-d} \int_0^\infty dx\; x^{\frac{d}{2}-1} \frac{\partial}{\partial t} P^{-1}
\end{aligned} \tag{12}
$$

depend on the precise form of the infrared cutoff R_k appearing in

$$
P(x) = x + Z_k^{-1} R_k(x)\ . \tag{13}
$$

In four dimensions one has $a_4 = 1,\ v_4 = 1/32\pi^2$ and eq. (10) reproduces the one-loop result for β_{g^2} in lowest order g^4. For the choice of R_k of [6] where $b_4 = 1$ an expansion of eq. (10) in powers of g^2 also gives 93 % of the perturbative two-loop coefficient. We observe that the approximations leading to (10) are valid only for $|\eta_F| < 1$. For larger values of $|\eta_F|$ we use a rougher estimate where b_d is set to zero.

Concerning the high-temperature field theory we should use the three-dimensional β-function for $k < k_T$, where $k_T = 2\pi T$ is the scale where the three-dimensional running sets in. The "initial value" of the gauge coupling reads $g^2(k_T) = 2\alpha_w(k_T)$ with $\alpha_w \approx \frac{1}{30}$ the four-dimensional weak fine structure constant. For $k < k_T$ the three-dimensional gauge coupling increases with a power behavior instead of the four-dimensional logarithmic behavior. The three-dimensional confinement scale $\Lambda_{\rm conf}^{(3)}$ - where g^2 diverges - is proportional to the temperature. Similarly, we may define (somewhat arbitrarily) the scale $k_{\rm np}$ where nonperturbative effects become important by $|\eta_F(k_{\rm np})| = 1$. For the electroweak theory and the choices (13) $P(x) = x + k^2$ ($P(x) = x/(1 - \exp -\frac{x}{k^2})$) one finds [6]

$$
\begin{aligned}
\Lambda_{\rm conf}^{(3)} &= 0.14T \quad (0.12T) \\
k_{\rm np} &= 0.35T \quad (0.31T)\ .
\end{aligned} \tag{14}
$$

For the symmetric phase of the electroweak theory one therefore has to deal with a strongly interacting gauge theory with typical nonperturbative mass scales only somewhat below the temperature scale! Similar to QCD one expects that condensates like $< F_{ij} F^{ij} >$ play an important role [6-8]. More generally, the physics of the symmetric phase corresponds to a strongly coupled $SU(2)$ Yang-Mills theory in three dimensions: The relevant excitations are "W-balls" (similar to glue balls or strongly interacting W-bosons) and scalar bound states. All "particles" are massive (except the "photon") and the relevant mass scale is set by $\Lambda_{\rm conf}^{(3)} \sim T$. For the W-boson masses we expect typical masses between $k_{\rm np}$ and $\Lambda_{\rm conf}^{(3)}$ (cf. (14)). Also the values of all condensates are given by appropriate powers of the temperature. Since the temperature is the only scale available the energy density must have the same T-dependence as for an ideal gas

$$
\rho = cT^4 \tag{15}
$$

Only the coefficient c should be different from the value obtained by counting the perturbative degrees of freedom[5]. We expect that quarks and leptons form $SU(2)$ singlet bound states similar to the mesons in QCD[6]. A chiral condensate seems, however, unlikely in the high-temperature regime and we do not think that fermions play any important role for the dynamics of the electroweak phase-transition. The "photon" (or rather the gauge boson associated to weak hypercharge) decouples from the W-balls. Its effective high temperature coupling to fermion and scalar bound states is renormalized to a very small value. As for the phase with spontaneous symmetry breaking, the fermions and "photon" can be neglected for the symmetric phase. We conclude that the high-temperature phase-transition of the electroweak theory can be described by an effective three-dimensional Yang-Mills-Higgs system. It is strongly interacting in the symmetric phase. Depending on the value of the mass of the Higgs boson it may also be strongly interacting in the phase with spontaneous symmetry breaking if the temperature is near the critical temperature. A more detailed investigation of this issue will be given in the next section.

4. Renormalization Group improved Effective Potential for the Electroweak Phase-Transition

For an approximate study of the electroweak phase-transition we will now investigate a three-dimensional $SU(2)$-Higgs model [8]. It is related to the full electroweak theory at high temperatures by setting the $U(1)_Y$-coupling to zero and integrating out the non-static modes of all fields as well as all modes of the 0-component of the gauge-field. For an extensive discussion of this procedure, see [13]. We will work with the truncation

$$\begin{aligned}\Gamma_k\left[\varphi, A_\mu, \bar{A}_\mu\right] \;&=\; \int d^dx \Big(U_k(\rho) + Z_{\varphi,k}|D_\mu\varphi|^2 + \frac{1}{4} Z_{F,k} F_{\mu\nu}F^{\mu\nu} \\ &\quad + \frac{1}{2\alpha} Z_{F,k} \left(D_\mu[\bar{A}](A^\mu - \bar{A}^\mu)\right)^2 \Big)\end{aligned} \tag{16}$$

and a simple truncation in the ghost sector [6]. Here $\rho = \varphi^\dagger\varphi$ and group indices are omitted. The form of the average potential $U_k(\rho)$ is left arbitrary and has to be determined by solving the flow equation. For the Abelian Higgs model the evolution equation for the average potential was computed in the approximation (16) in reference [6]. Inserting the appropriate $SU(2)$ group factors we obtain in the Landau gauge ($\alpha = 0$), with $t = \ln k$

$$\begin{aligned}\frac{\partial}{\partial t} U_k(\rho) \;&=\; \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} \frac{\partial}{\partial t}\Big(6 \ln\left(q^2 + k^2 + m_B^2\right) + \\ &\quad + \ln\left(q^2 + k^2 + m_1^2\right) + 3\ln\left(q^2 + k^2 + m_2^2\right)\Big)\end{aligned} \tag{17}$$

[5]A similar remark also applies to high temperature QCD. We expect quantitative modifications of early cosmology due to the difference between c and the ideal gas value.

[6]We use here a language appropriate for the excitations of the three-dimensional Euclidean theory. Interpretation in terms of relativistic particles has to be used with care!

where the mass terms read

$$m_B^2 = \frac{1}{2} Z_{\varphi,k} \bar{g}_3^2 \rho \ ; \ m_1^2 = \left(U_k'(\rho) + 2\rho U_k''(\rho) \right) / Z_{\varphi,k} \ ; \ m_2^2 = U_k'(\rho)/Z_{\varphi,k}. \tag{18}$$

We have used here a masslike infrared cutoff $R_k = Z_k k^2$. The partial derivative $\frac{\partial}{\partial t}$ on the right hand side of (17) is meant to act only on R_k and we omit contributions arising from the wave function renormalization Z_k in R_k. Primes denote derivatives with respect to ρ.

This flow equation[7] constitutes a nonlinear partial differential equation for the dependence of U on the two variables k and ρ. In our case it holds for the three dimensional potential U_3 and a correspondingly normalized scalar field ρ_3. They are related to the usual four dimensional quantities by $U_3 = U_4/T$, $\rho_3 = \rho_4/T$. Equation (17) is the basic equation of this section and has to be supplemented by corresponding equations for the k-dependence of $\bar{g}_3^2$ and Z_φ. For given initial conditions at k_T we aim for a solution for $k \to 0$ in order to compute the free energy U_0. The gauge coupling in (18) stands for the three dimensional running renormalized gauge coupling $\bar{g}_3^2(k)$. Its value at the scale k_T is given by

$$\bar{g}_3^2(k_T) = g_4^2(k_T) T \left(1 - \frac{g_4^2(k_T) T}{24\pi m_D} \right) \tag{19}$$

where

$$m_D^2 = \frac{5}{6} g_4^2(k_T) T^2 \tag{20}$$

accounts for the effects of integrating out the A_0 mode in lowest order [13]. The evolution equation for the running gauge coupling in the pure Yang-Mills theory has been given above (eq. (10)), and reads in lowest order

$$\frac{\partial}{\partial t} \bar{g}_3^2 = \beta_{g^2} = -\frac{23\tau}{24\pi} \bar{g}_3^4(k) k^{-1} \ . \tag{21}$$

The deviation of τ from one accounts for the small contributions of scalar fluctuations which are not included here[8]. Equation (21) is easily solved,

$$\frac{1}{\bar{g}_3^2(k)} = \frac{1}{\bar{g}_3^2(k_T)} + \frac{23\tau}{24\pi} \left(\frac{1}{k_T} - \frac{1}{k} \right) \tag{22}$$

[7] The ultraviolet divergence on the right hand side of equation (17) is particular to the use of a masslike infrared cutoff. For $d = 3$ it concerns only an irrelevant constant in U and is absent for $\partial U'/\partial t$.

[8] For a suitable choice of wave function renormalization constants in the infrared cutoff for the gauge bosons the lowest order result becomes independent of the gauge parameter α and can therefore be used for the Landau gauge employed here.

and yields the confinement scale (cf. eq.(14)) in lowest order

$$\Lambda^{(3)}_{\rm conf} = \left(\frac{1}{k_T} + \frac{24\pi}{23\tau\bar{g}^2_3(k_T)}\right)^{-1} . \tag{23}$$

Furthermore, we will need the anomalous dimension of the scalar field. For our purpose it can be approximated by [6]

$$\eta_\varphi = -\frac{\partial \ln Z_\varphi}{\partial t} = -\frac{1}{4\pi}\bar{g}^2_3(k)k^{-1} \tag{24}$$

and we set $Z_\varphi = 1$ for $k = k_T$.

A convenient quantity for an investigation of the effective potential is the ρ-dependent quartic coupling

$$\bar{\lambda}_{3,k}(\rho) = U''_k(\rho) = \frac{\partial^2 U_{3,k}}{\partial {\rho_3}^2}. \tag{25}$$

Knowing for $k = 0$ the function $\bar{\lambda}_3(\rho) = \bar{\lambda}_{3,0}(\rho)$, the high temperature effective potential $U(\rho) = U_0(\rho)$ can be reconstructed by integration and translation to a four dimensional normalization. One of the two integration constants is irrelevant and the other (the mass term linear in ρ) can be found by adapting $U(\rho)$ to the perturbative result for large ρ where the three-dimensional running of the couplings is irrelevant.

The evolution equation for the k-dependence of $\bar{\lambda}_3(\rho_3)$ can be inferred from (17) by differentiating twice with respect to ρ_3 and reads

$$\frac{\partial\bar{\lambda}_{3,k}(\rho_3)}{\partial t} = \frac{3}{32\pi}\left(\frac{Z^2_{\varphi,k}\bar{g}^4_{3,k}k^2}{(k^2+m^2_B)^{3/2}} + \frac{6Z^{-2}_{\varphi,k}\bar{\lambda}^2_{3,k}(\rho_3)k^2}{(k^2+m^2_1)^{3/2}} + \frac{2Z^{-2}_{\varphi,k}\bar{\lambda}^2_{3,k}(\rho_3)k^2}{(k^2+m^2_2)^{3/2}}\right) \tag{26}$$

where we have neglected terms $\propto U^{(3)}_k(\rho_3)$ and $U^{(4)}_k(\rho_3)$. We report here on an approximate solution of the flow equation for $\bar{\lambda}_3(\rho_3)$ for $k = 0$ [8]. It is based on the observation that the ρ_3-dependent mass terms m^2_B, m^2_1, and m^2_2 act in equation (17) as independent infrared cutoffs in just the same way as k^2. A variation of m^2 for $k^2 = 0$ is roughly equivalent to a variation of k^2 at $m^2 = 0$. We use this observation to translate the flow equation (26) into a renormalization group equation for $\bar{\lambda}_3(\rho_3)$ at $k = 0$: In equation (26) we replace $\frac{\partial}{\partial t}$ by $\frac{\partial}{\partial t'} = m_B\frac{\partial}{\partial m_B}$ and the factors $k^2\left(k^2+m^2\right)^{-3/2}$ by m^{-1}. We can then work with a new effective infrared cutoff $k' = m_B$ which is a function of ρ_3 (we omit the prime on k in the following),

$$k^2 = m^2_B = \frac{1}{2}Z_\varphi(k)\bar{g}^2_3(k)\rho_3. \tag{27}$$

This procedure transforms equation (26) into a simple differential equation for $\bar{\lambda}_3(\rho_3) = \bar{\lambda}_3(k(\rho_3))$. In terms of the renormalized coupling

$$\bar{\lambda}_R(k) = Z^{-2}_\varphi(k)\bar{\lambda}_3(k) \tag{28}$$

it reads[9]

$$\frac{\partial}{\partial t}\bar{\lambda}_R(k) = \frac{3}{32\pi k}\left(\bar{g}_3^4(k) + \left(\sqrt{6}+\sqrt{2}\right)\bar{\lambda}_R^{3/2}(k)\bar{g}_3(k)\right) - \frac{1}{2\pi k}\bar{g}_3^2(k)\bar{\lambda}_R(k). \tag{29}$$

For the terms $\propto \frac{1}{m_1}$ and $\propto \frac{1}{m_2}$ from equation (26) we have approximated in (18) $U'(\rho_3) \simeq \rho_3\bar{\lambda}_R(\rho_3)$ which amounts to neglecting the mass term. For negative $\bar{\lambda}_R(k)$ our approximation does not describe properly the effect of the scalar fluctuations. Since their contribution is small in this region we simply omit the terms $\propto \bar{\lambda}_R^{3/2}$ once $\bar{\lambda}_R(k)$ becomes negative.

In order to solve the flow equation (29) we need an initial value $\bar{\lambda}_3(k_T)$, in addition to (19). This will depend on the (zero temperature) scalar mass M_H. Furthermore, the ρ-integration of (25) leads to a term $-\mu^2(T)\rho_3$ with the temperature dependent mass term $\mu^2(T)$ as an integration constant. The values of $\bar{\lambda}_3(k_T)$ and $\mu^2(T)$ describe how a given four-dimensional model at $T > 0$ translates into an equivalent three-dimensional one. They can be fixed by the observation that for large ρ, such that $m_B^2(\rho) > k_T^2$, the one-loop expression for the effective potential [26]

$$\tilde{U}_3(\rho_3) = -\mu^2(T)\rho_3 + \frac{1}{2}\left(\bar{\lambda}_3 + \Delta\bar{\lambda}_3\right){\rho_3}^2 - \frac{1}{12\pi}\left(6m_B^3 + 3m_E^3 + \bar{m}_1^3 + 3\bar{m}_2^3\right) \tag{30}$$

should be a good approximation. Here the masses are given by $\bar{m}_1^2 = 3\bar{\lambda}_3\rho_3 - \mu^2(T)$, and $\bar{m}_2^2 = \bar{\lambda}_3\rho_3 - \mu^2(T)$, and we set $Z_\varphi = 1$. Also the correction

$$\Delta\bar{\lambda}_3 = \frac{3\bar{g}_3^4}{64\pi^2 T}\left(1 + \frac{\sqrt{6}+\sqrt{2}}{8}\frac{M_H^3}{M_H^3}\right) \tag{31}$$

is chosen such that $\bar{\lambda}_3 = \tilde{U}_3''(8\pi^2T^2/\bar{g}_3^2(k_T))$. High temperature perturbation theory to one loop with the A_0 integrated out leads then to the initial value

$$\bar{\lambda}_3(k_T) = \frac{1}{4}g_4^2(k_T)T\frac{M_H^2}{M_W^2} - \frac{3g_4^4(k_T)T^2}{64\pi m_D} - \Delta\bar{\lambda}_3\ . \tag{32}$$

We observe that the inclusion of two-loop effects or fermions will change the relation between $\bar{\lambda}_3(k_T)$ and M_H. This leads to a rescaling between the scalar mass quoted in this work and the true physical mass. The mass term $\mu^2(T)$ has in two-loop perturbation theory the genuine temperature dependence

$$\mu^2(T) = \beta M_H^2 - \gamma T^2 + \bar{\gamma}T^2\ln\frac{T}{M_W}\ . \tag{33}$$

Here β and γ are independent of T [10]. They depend, however, on the regularization scheme and this is particularly important for γ as indicated already by a possible

[9] A more formal justification for eq. (29) can be found in [8].

[10] To one loop order one has (A_o integrated out, no quarks, $\alpha_w = \frac{g_4^2(k_T)}{4\pi}$)

$$\beta = \frac{1}{2}\ ,\ \gamma = \frac{\pi}{4}\alpha_w\left(3 - \frac{3m_D}{\pi T} + \frac{M_H^2}{M_W^2}\right)$$

and no $T^2\ln T$ term.

change of scale in the $\ln T$-term. Some care is needed for the proper comparison between three- and four-dimensional lattice regularizations, high temperature perturbation theory in various versions and our renormalization group approach. The most reliable way of comparison seems to us to equate renormalized quantities in the various approaches. A good candidate for fixing β and γ seems to be the expectation value $\Phi_0(T)$ at two different temperatures T_1 and T_2 [11].

Combining equations (25), (27), (28) and (29) with flow equations for $\bar{g}_3^2$ and η_φ, we can compute the ρ_3-dependence of the high temperature effective potential as a solution of the flow equation. It is interesting to note that except for the last term arising from the anomalous dimension, eq. (29) can also be directly obtained by taking appropriate derivatives of the one loop formula (30), treating mass ratios such as m_B/m_1 as k-independent and replacing at the end the couplings $\bar{g}_3^2$ and $\bar{\lambda}_3$ by running couplings evaluated at the scale k. For $Z_\varphi = 1$ and $\bar{g}_3^2$, $\bar{\lambda}_3$ independent of k equation (29) reproduces exactly the one loop result. Our renormalization group improvement enables us to include the effects of running couplings and the anomalous dimension. This should reproduce a large part of the two-loop corrections and also higher contributions. The main changes as compared to the 1-loop calculations can be understood from the corresponding differential equations: The inclusion of η_φ lowers the scale at which U'' changes sign, whereas the running of the gauge coupling acts the opposite way. Thus at a given k, corresponding to a given ρ_3, the running of $\bar{g}_3^2$ makes the potential bend up less than the 1-loop calculation predicts. The inclusion of the term $\propto \bar{\lambda}_R^{3/2} \bar{g}_3$ will have the same effect but is quantitatively important only for a large scalar mass.

Translated to the effective potential the running of $\bar{g}_3^2$ should strengthen the phase-transition and lower the critical temperature as compared to the one-loop result. This is what one would naively expect, since the first order character of the transition is due to the gauge-boson loops. Thus, enhancing the coupling should give a transition more strongly first order. On the other hand, non-perturbative condensation phenomena may have the opposite effect. Also the inclusion of the anomalous dimension η_φ weakens the transition. To estimate the precise effects of the running of the couplings on the shape of the potential we proceed to a numerical investigation of equations (10), (24), and (29). (For all numerical work we use $g_4 = 2/3$, $M_w = 80.6$ GeV, and $\tau = 1$.) Additional effects from condensation phenomena will be qualitatively discussed and added later. In fig.s 2, 3 and 4 we show the effective potential as obtained with our method for different temperatures and masses of the Higgs scalar. In all plots we use four dimensional quantities and plot $\delta U = U_4(\Phi) - U_4(0)$ versus

[11] Here $T_{1,2}$ should be sufficiently below the critical temperature such that the potential minimum occurs in a region of ρ where two-loop perturbation theory is reliable. For purposes of comparison with three-dimensional lattice results we have chosen β and γ such as to obtain the same $\Phi_0(T_{1,2})$ as in two-loop perturbation theory, with T_1 and T_2 in a region where lattice data and perturbation theory are in good agreement for the prediction of the location of the potential minimum.

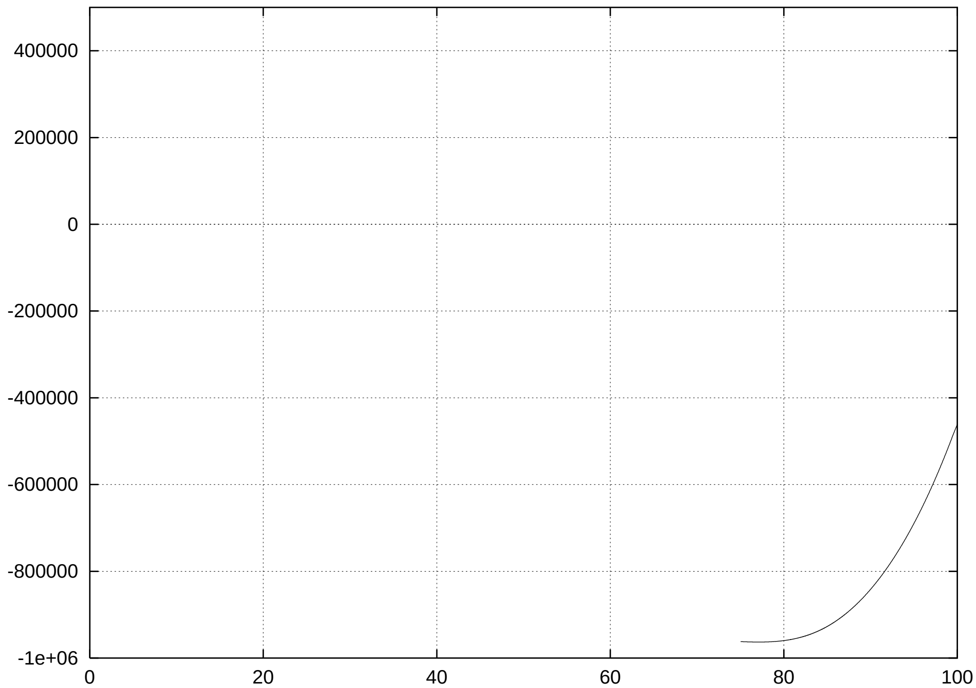

Figure 2: The effective potential for $M_H = 80$ GeV with possible contributions from condensation phenomena. All curves are at their respective critical temperature. The curve denoted by (1) is the potential without nonperturbative effects ($a = 0$, $T_c = 172.8$ GeV). The other curves correspond to the critical temperature obtained by lattice simulations [16], $T_c = 167.7$ GeV, and the following values of the parameters for the nonperturbative effects:
(2) - $a = 0.2928$, $b = 3$, $c = 1.1$, $\alpha = 1$ (solid line);
(3) - $a = 0.2885$, $b = 3$, $c = 1$, $\alpha = 2$ (long dashed line);
(4) - $a = 0.305$, $b = 1$, $c = 1$, $\alpha = 1$ (short dashed line);
(5) - $a = 0.3005$, $b = 1$, $c = 1$, $\alpha = 2$ (dotted line).
The lower dashed-dotted line gives the perturbative potential at the lattice-critical temperature.

$\Phi = \sqrt{\rho_4}$. The first dashed-dotted curve in figure 2 (denoted by (1)) corresponds to the critical temperature which would be obtained from our renormalization group-improved approach, neglecting condensation effects. The second dashed-dotted curve in figure 2 gives the analogous result for a temperature corresponding to the critical temperature inferred from lattice simulations [16]. We will now discuss possible alterations due to nonperturbative condensates and demonstrate how they could lead to agreement between our method and lattice results.

The effective potential shown in figures 2 through 4 is expected to be rather reliable for large enough values of Φ where the effective gauge coupling is not yet too

strong. On the other hand, the truncation (16) becomes insufficient for large g and we expect important modifications for $\Phi < \Phi_{\rm np}$, where (eq.s (14) and (27))

$$\begin{aligned} \Phi_{\rm np} &= \left(\frac{2k_{\rm np}^2 T}{Z_\varphi(k_{\rm np})\bar{g}_3^2(k_{\rm np})}\right)^{\frac{1}{2}} \\ |\eta_F(k_{\rm np})| &= 1\ . \end{aligned} \tag{34}$$

In fact, the flow equations (10), (26) do not account for effects like the W-condensation mentioned in the preceding section. An easy way to visualize the relevance of such effects is the introduction of a composite $SU(2)$-singlet field χ for the description of condensation phenomena in the gauge sector. For example, one may choose $\chi \propto F_{ij}F^{ij}$ or some other (properly regularized) operator. Condensation phenomena are then described by the vacuum expectation value χ_0. With an appropriate normalization of χ they give a contribution to the free energy

$$\Delta U_3 = \chi_0^3\ ,\ \ \Delta U = \chi_0^3 T\ . \tag{35}$$

For $\Phi = 0$ the only available scale is $k_{\rm np} \propto T$, and therefore $\chi_0 = a k_{\rm np}$. The dimensionless coefficient a is expected of order one since the gauge coupling grows very rapidly for $k < k_{\rm np}$ and there is not much difference between $k_{\rm np}$ and $\Lambda_{\rm conf}^{(3)}$ (c.f. eq. (14)). It is also clear that the value of χ_0 must depend on Φ: For large Φ condensation phenomena should be essentially absent since the gauge coupling remains small. We will parameterize the Φ-dependent condensation effects by an additional contribution to the free energy

$$\Delta U = -\left[a k_{\rm np} f\left(\frac{g^2(k)}{g_{\rm np}^2}\right)\right]^3 T \tag{36}$$

where f describes how χ_0 depends on the effective gauge-coupling, with $f(z\to\infty) = 1$ and $f(z)$ vanishing rapidly for $z \ll 1$. As an example we take

$$f(z) = \frac{2}{\pi}\arctan\left(\frac{\pi}{2}b(z-c)^\alpha\right) \tag{37}$$

for $z > c$ and $f = 0$ otherwise. Here c indicates for which g^2 the condensation sets in, and b is a measure how fast the condensation phenomena build up. The Φ-dependence of ΔU arises indirectly through the Φ-dependence of g via the identification (27).

Condensation phenomena lower the free energy around the origin ($\Phi = 0$) and therefore lead to a lower critical temperature [7, 8]. This is consistent with the fact that the critical temperature computed without ΔU comes out systematically higher than found in lattice simulations. One may even use the lattice results for T_c to give a rough estimate of the coefficient a. Employing (36),(37) with $b = c = 1$ we have adapted a such that the critical temperature coincides with the central values

from lattice simulations. This yields $a \sim 0.3$ for the simulation of [16] at $M_H = 80$ GeV. An attempt for an estimate of the size of condensation effects for $M_H = 35$ GeV yields $a \sim 0.38$ [17]. Since the condensation effects are mainly related to the gauge field degrees of freedom and also the scalar contribution to the running of g^2 is small one expects a to be independent of M_H in a first approximation [8]. This conjecture seems to be consistent within the large uncertainties of the quoted values.

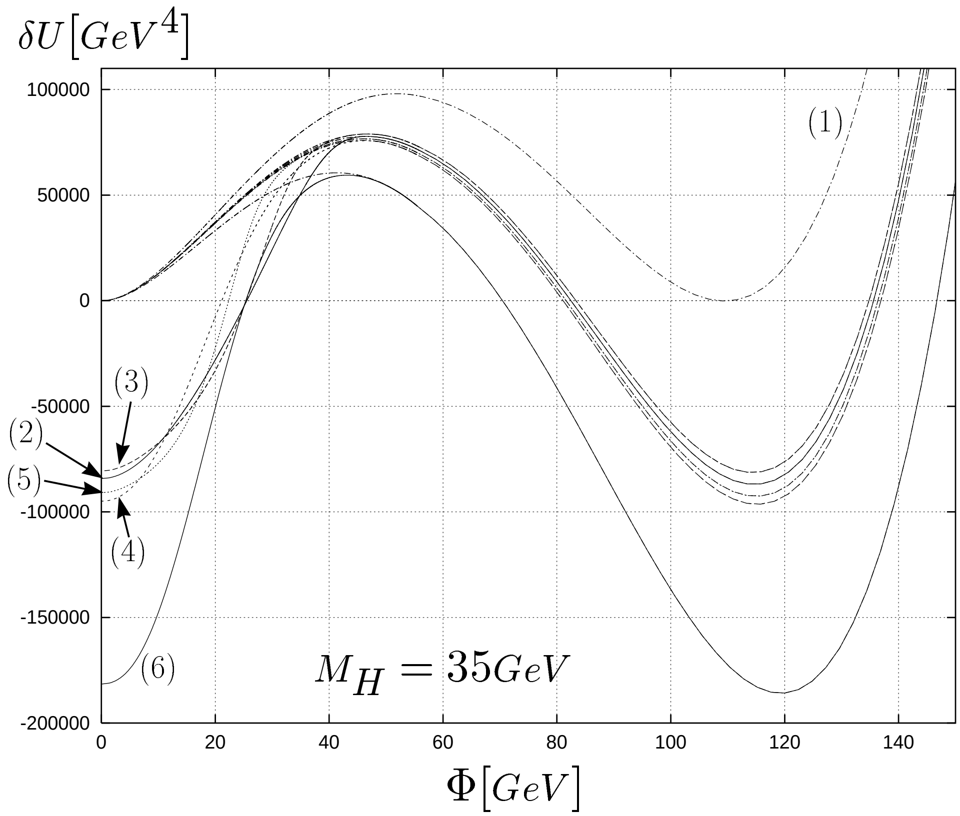

Figure 3: As figure 2, but with $M_H = 35$ GeV.
(1) - $a = 0$, $T = 94.61$ GeV (dashed-dotted line);
(2) - $a = 0.2928$, $b = 3$, $c = 1.1$, $\alpha = 1$, $T = 94.13$ GeV (solid line);
(3) - $a = 0.2885$, $b = 3$, $c = 1$, $\alpha = 2$, $T = 94.16$ GeV (long dashed line);
(4) - $a = 0.305$, $b = 1$, $c = 1$, $\alpha = 1$, $T = 94.08$ GeV (short dashed line).
(5) - $a = 0.3005$, $b = 1$, $c = 1$, $\alpha = 2$, $T = 94.10$ GeV (dotted line).
The curve denoted by (6) corresponds to the value of a as obtained from lattice simulations for $M_H = 35$ GeV [17]:
(6) - $a = 0.381$, $b = 1$, $c = 1$, $\alpha = 1$, $T = 93.62$ GeV (solid line).

For $\Phi = 0$ we note that χ_0 roughly equals the three-dimensional confinement scale for $a \sim 0.3 - 0.4$. Comparison with the curves for $a = 0$ (dashed line in figure 2) demonstrates the importance of the nonperturbative phenomena for $M_H = 80$ GeV. Our lack of quantitative knowledge of the condensation phenomena is reflected by the difference of the curves for $\alpha = 1$ and 2 and $b = 1$ and 3. For $\alpha = 2$ (curves (3)

and (5) in figure 2), ΔU gives no contribution to the massterm at the origin and we observe the minimum at $\Phi > 0$ even for the "symmetric" phase. This would lead to an effective magnetic mass as described earlier in the context of gap-equations [18].

For $M_H = 80$ GeV we observe that condensation phenomena may weaken or strengthen the phase transition, depending on the values of α and b. As a general tendency we observe that for a fast onset of the condensation (large b) the phase transition becomes weaker than one would expect from perturbation theory (compare curves (1) and (2)). In fig. 3 we give for comparison the potential at the critical temperature for $M_H = 35$ GeV, with the same parameters a, b, c, and α as in figure 2. In addition, we also present a curve for $a = 0.381$ (denoted as (6) in the figure) [17]. For such a low value of the scalar mass one expects perturbation theory to be rather

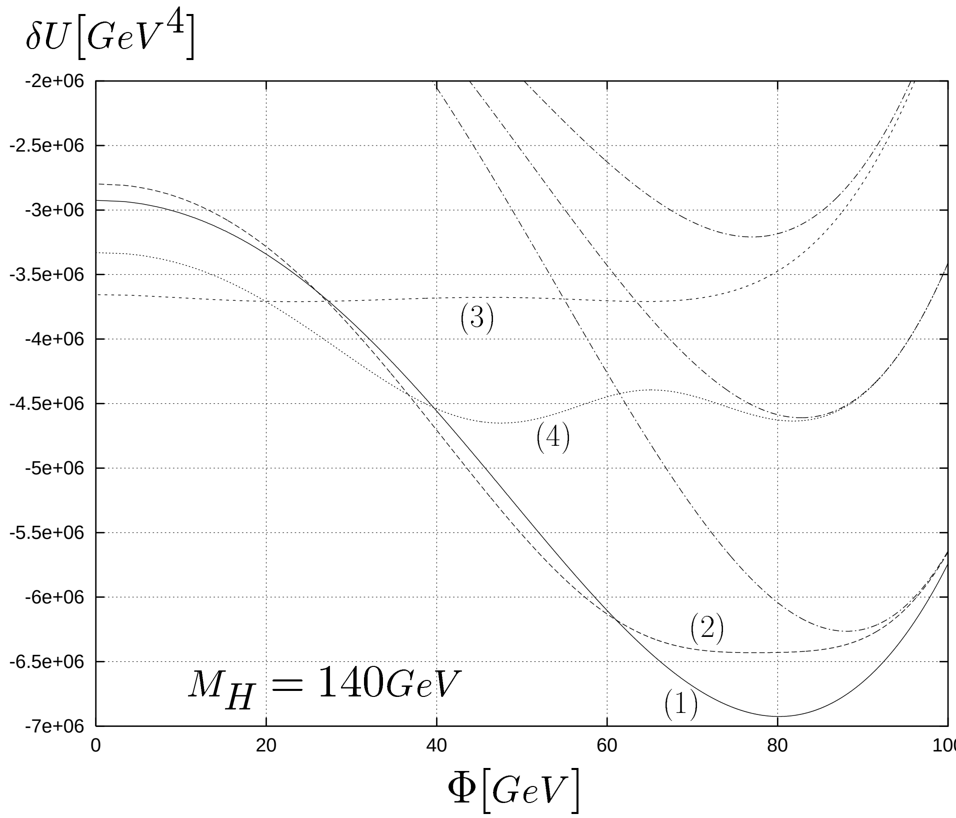

Figure 4: As figure 2, but for $M_H = 140$ GeV. Here only the curves corresponding to the critical temperatures with nonvanishing condensation effects are given. The dashed-dotted lines give the potential at the respective temperatures neglecting condensation phenomena. The parameters are:
(1) - $a = 0.2928$, $b = 3$, $c = 1.1$, $\alpha = 1$, $T = 228.5$ GeV (solid line);
(2) - $a = 0.2885$, $b = 3$, $c = 1$, $\alpha = 2$, $T = 228.5$ GeV (long dashed line);
(3) - $a = 0.305$, $b = 1$, $c = 1$, $\alpha = 1$, $T = 234.3$ GeV (short dashed line).
(4) - $a = 0.3005$, $b = 1$, $c = 1$, $\alpha = 2$, $T = 231.5$ GeV (dotted line).
For the choices of parameters corresponding to (1) and (2), the transition has changed to an analytical crossover.

reliable. This is confirmed by the fact that the critical temperature T_c and the location of the minimum $\Phi_0(T_c)$ depend only very moderately on the condensation effects. Only the barrier height, related to the surface tension, depends substantially on a, being nevertheless almost independent of the precise form of f as encoded in b, c, and α. For a fixed temperature the condensation effects influence the shape of the potential only for $\Phi \lesssim 40$ GeV, far away from the location of the minimum. The strength of the transition increases with a, independent of the shape of f.

Finally, we also try an extrapolation to larger values of the scalar mass, as shown in figure 4 for $M_H = 140$ GeV. At the critical temperature the shape of the potential now depends very strongly on the shape of f and an understanding of the condensation phenomena becomes crucial even for the qualitative picture. We observe that all curves for $a \sim 0.3$ have for the symmetric phase a minimum at $\Phi_0 > 0$. As b is increased, this minimum moves toward the minimum corresponding to the phase with spontaneous symmetry breaking. For the curve with $\alpha = 1$, $b = 3$ the two minima have already melted and there remains no true phase transition (solid line). For this form of f one would predict an analytical crossover for $M_H = 140$ GeV. This illustrates the speculation [6] that, as a function of M_H, the critical line corresponding to the first order transition ends at some value $M_{H,c}$. For this critical value of the scalar mass the phase transition would have to be of second order[12], with vanishing scalar mass at the critical temperature. By accident, this situation is realized approximately for $\alpha = 2$, $b = 3$ (i.e. $M_{H,c} \sim 140$ GeV in this case). It would be very interesting to know the true value of the critical Higgs-mass!

Comparing figures 2, 3, and 4 we have learned that the importance of nonperturbative condensation effects strongly increases with M_H. We conclude that perturbation theory can only give a realistic description of the phase transition for a small scalar mass, whereas the nonperturbative effects are of crucial importance for the understanding of the electroweak phase transition for realistic values of the scalar mass.

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